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A new multiple attribute decision making method based on linear programming methodology and novel score function and novel accuracy function of interval-valued intuitionistic fuzzy values

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ABSTRACT

Score functions and accuracy functions of interval-valued intuitionistic fuzzy values (IVIFVs) play important roles in dealing with multiple attribute decision making (MADM) problems in interval-valued intuitionistic fuzzy (IVIF) environments. In this paper, we propose a new MADM method using the linear programming (LP) methodology and the proposed new score function and the proposed new accuracy function of IVIFVs for overcoming the drawbacks of Wang and Chen's MADM method (2017), which has the drawbacks that the preference order (PO) of alternatives cannot be distinguished in some cases and it gets an infinite number of solutions of the optimal weights of attributes when the summation values of some columns in the transformed decision matrix (TDM) are the same, such that it obtains different POs of alternatives.

1. Introduction

Some researchers have presented multiple attribute decision making (MADM) methods [2–7,22,24,26,27] using interval-valued intuitionistic fuzzy sets (IVIFSs) [1]. In [2], Bai proposed an interval-valued intuitionistic fuzzy (IVIF) TOPSIS method to deal with MADM problems based on a score function. In [7], Garg presented a generalized improved score function and proposed a multi-criteria decision making method with unknown attribute weights under interval-valued intuitionistic fuzzy (IVIF) environments. In [22], Tu and Chen presented two score functions for IVIFVs for dealing with multi-criteria decision making analysis problems. In [24], Wang and Chen proposed a MADM method using the linear programming (LP) methodology [4,21] and a score function of IVIFVs. In [26], Wang et al. presented a MADM method with IVIF assessments and incomplete weights. In [27], Xu presented methods for aggregating IVIF information for MADM.

However, Wang and Chen's MADM method [24] has the shortcomings that (1) it cannot distinguish the preference order (PO) of alternatives in some cases due to the fact Wang and Chen's score function [24] of IVIFVs has the shortcoming that it cannot distinguish IVIFVs in some cases and (2) the linear LP model constructed in Wang and Chen's MADM method [24] has the shortcoming that it will get an infinite number of solutions of the optimal weights of attributions when the summation values of some columns in the transformed decision matrix (TDM) are the same. As a result, Wang and Chen's MADM method has the drawback that it gets different preference orders (POs) of alternatives when the summation values

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of some columns in the TDM are the same. Therefore, we need to propose a new MADM method in IVIF environments to overcome the shortcoming of Wang and Chen's MADM method [24].

In this paper, we propose a new method to deal with MADM problems in IVIF environments using the LP methodology [4,21] and the proposed new score function and new accuracy function of IVIFVs for overcoming the shortcomings of Wang and Chen's MADM method [24]. The proposed MADM method offers us a very useful way to deal with MADM problems under IVIF environments.

The rest of this paper is organized as follows. In Section 2, we briefly review basic concepts of IVIFSs [1], IVIFVs [27], Wang and Chen's score function [24] of IVIFVs and the interval-valued intuitionistic fuzzy weighted averaging (IVIFWA) operators of IVIFVs presented in [25] and [27], respectively. We also propose a new score function and a new accuracy function of IVIFVs. Moreover, we also propose a new ranking method of IVIFVs based on the proposed new score function and the proposed new accuracy function of IVIFVs. In Section 3, we point out the drawbacks of Wang and Chen's MADM method [24] and use some examples to illustrate the drawbacks of Wang and Chen's MADM method [24]. In Section 4, we propose a new MADM method using the LP methodology and the proposed new score function and the proposed new accuracy function of IVIFVs. The conclusions are discussed in Section 5.

2. Preliminaries

In this section, we briefly review basic concepts of IVIFSs [1], IVIFVs [27], Wang and Chen's score function [24] of IVIFVs and the IVIFWA operators of IVIFVs presented in [25] and [27], respectively. We also propose a new score function and a new accuracy function of IVIFVs and propose a new ranking method of IVIFVs based on the proposed new score function and the proposed new accuracy function of IVIFVs.

Definition 2.1 [1]. Let $X = \{x_1, x_2, \ldots, x_n\}$ be the universe of discourse. An IVIFS $\tilde{A}$ in X is represented by $\tilde{A} = \{< x_i, \mu_{\tilde{A}}(x_i), \upsilon_{\tilde{A}}(x_i) > | x_i \in X\}$, where $1 \le i \le n$, $\mu_{\tilde{A}}(x_i)$ and $\upsilon_{\tilde{A}}(x_i)$ denote the membership degree and the non-membership degree of element x_i belonging to the IVIFS $\tilde{A}$, respectively, $\mu_{\tilde{A}}(x_i) = [\mu^-_{\tilde{A}}(x_i),\ \mu^+_{\tilde{A}}(x_i)]$, $\upsilon_{\tilde{A}}(x_i) = [\upsilon^-_{\tilde{A}}(x_i), \upsilon^+_{\tilde{A}}(x_i)]$, $0 \le \mu^-_{\tilde{A}}(x_i) \le \mu^+_{\tilde{A}}(x_i) \le 1$, $0 \le \upsilon^-_{\tilde{A}}(x_i) \le \upsilon^+_{\tilde{A}}(x_i) \le 1$ and $\mu^+_{\tilde{A}}(x_i) + \upsilon^+_{\tilde{A}}(x_i) \le 1$. The hesitancy degree $\pi_{\tilde{A}}(x_i)$ of element x_i belonging to the IVIFS $\tilde{A}$ is represented by $\pi_{\tilde{A}}(x_i) = [\pi^-_{\tilde{A}}(x_i), \pi^+_{\tilde{A}}(x_i)]$, where $\pi^-_{\tilde{A}}(x_i) = 1 - \mu^+_{\tilde{A}}(x_i) - \upsilon^+_{\tilde{A}}(x_i)$, $\pi^+_{\tilde{A}}(x_i) = 1 - \mu^-_{\tilde{A}}(x_i) - \upsilon^-_{\tilde{A}}(x_i)$ and $1 \le i \le n$.

In [27], the pair $([\mu^-_{\tilde{A}}(x_i),\ \mu^+_{\tilde{A}}(x_i)],\ [\upsilon^-_{\tilde{A}}(x_i), \upsilon^+_{\tilde{A}}(x_i)])$ is called an IVIFV, where $0 \le \mu^-_{\tilde{A}}(x_i) \le \mu^+_{\tilde{A}}(x_i) \le 1$, $0 \le \upsilon^-_{\tilde{A}}(x_i) \le \upsilon^+_{\tilde{A}}(x_i) \le 1$ and $\mu^+_{\tilde{A}}(x_i) + \upsilon^+_{\tilde{A}}(x_i) \le 1$ and $1 \le i \le n$.

Definition 2.2 [24]. Let $\tilde{\alpha} = ([a^-,\ a^+],\ [b^-,\ b^+])$ be an IVIFV, where $[a^-,\ a^+] \subseteq [0,\ 1]$, $[b^-,\ b^+] \subseteq [0,\ 1]$ and $a^+ + b^+ \le 1$. Wang and Chen's score function $S_{WC}(\tilde{\alpha})$ of the IVIFV $\tilde{\alpha}$ is defined as follows:

$$S_{WC}(\tilde{\alpha}) = \frac{a^- + a^+ + \sqrt{a^+ b^+}(1 - a^- - b^-) + \sqrt{a^- b^-}(1 - a^+ - b^+)}{2}, \tag{1}$$

where $S_{WC}(\tilde{\alpha}) \in [0,\ 1]$. The larger the score value $S_{WC}(\tilde{\alpha})$ of the IVIFV $\tilde{\alpha}$, the larger the IVIFV $\tilde{\alpha}$.

However, Wang and Chen's score function S_{WC} [24] of IVIFVs has the shortcoming that it cannot distinguish IVIFVs in some cases.

Example 2.1. Let $\tilde{\alpha}_1 = ([0.5914,\ 0.6383],\ [0.1266,\ 0.2429])$ and $\tilde{\alpha}_2 = ([0.5530,\ 0.6574],\ [0.1000,\ 0.2603])$ be two different IVIFVs. According to Eq. (1), we can obtain $S_{WC}(\tilde{\alpha}_1) = S_{WC}(\tilde{\alpha}_2) = 0.6866$. Thus, Wang and Chen's score function S_{WC} [24] shown in Eq. (1) cannot distinguish the IVIFVs $\tilde{\alpha}_1 = ([0.5914, 0.6383],\ [0.1266,\ 0.2429])$ and $\tilde{\alpha}_2 = ([0.5530,\ 0.6574],\ [0.1000,\ 0.2603])$ in this situation.

In this paper, we propose a new score function S_{NWC} and a new accuracy function H_{NWC} of IVIFVs to overcome the shortcomings Wang and Chen's score function S_{WC} [24] of IVIFVs.

Definition 2.3. Let $\tilde{\alpha} = ([a^-,\ a^+],\ [b^-,\ b^+])$ be an IVIFV, where $[a^-,\ a^+] \subseteq [0,\ 1]$, $[b^-,\ b^+] \subseteq [0,\ 1]$ and $a^+ + b^+ \le 1$. The proposed score function S_{NWC} of the IVIFV $\tilde{\alpha}$ is defined as follows:

$$S_{NWC}(\tilde{\alpha}) = \frac{(a^- + a^+)(a^- + b^-) - (b^- + b^+)(a^+ + b^+)}{2}, \tag{2}$$

where $S_{NWC}(\tilde{\alpha}) \in [-1,\ 1]$. The larger the score value $S_{NWC}(\tilde{\alpha})$ of the IVIFV $\tilde{\alpha}$, the larger the IVIFV $\tilde{\alpha}$.

The proposed score function S_{NWC} of IVIFVs has the following properties:

Property 2.1. If the IVIFV $\tilde{\alpha} = ([a^-,\ a^+],\ [b^-,\ b^+])$, where $[a^-,\ a^+] \subseteq [0,\ 1]$, $[b^-,\ b^+] \subseteq [0,\ 1]$ and $a^+ + b^+ \le 1$, then $S_{NWC}(\tilde{\alpha}) \in [-1,\ 1]$.

Proof. If the IVIFV $\tilde{\alpha} = ([1,\ 1],\ [0,\ 0])$, then based on Eq. (2), we can get

$$S_{NWC}(\tilde{\alpha}) = \frac{(1 + 1)(1 + 0) - (0 + 0)(1 + 0)}{2} = \frac{2}{2} = 1.$$

If the IVIFV $\tilde{\alpha} = ([0,\ 0],\ [1,\ 1])$, then based on Eq. (2), we can get

$$S_{NWC}(\tilde{\alpha}) = \frac{(0+0)(0+\ 1)\ -\ (1\ +\ 1)(0+1)}{2} = \frac{-2}{2} = -1.$$

Therefore, we can get $S_{NWC}(\tilde{\alpha}) \in [-1,\ 1]$. ■

Example 2.2. The same assumption as Example 2.1, where $\tilde{\alpha}_1 = ([0.5914,\ 0.6383],\ [0.1266,\ 0.2429])$ and $\tilde{\alpha}_2 = ([0.5530,\ 0.6574],\ [0.1000,\ 0.2603])$ are two different IVIFVs. Based on the proposed score function S_{NWC} shown in Eq. (2), we can get $S_{NWC}(\tilde{\alpha}_1) = 0.2787$ and $S_{NWC}(\tilde{\alpha}_2) = 0.2299$. Therefore, we can see that the proposed score function S_{NWC} shown in Eq. (2) has the advantage that it can overcome the drawback of Wang and Chen's score function S_{WC} [24] to distinguish the IVIFVs $\tilde{\alpha}_1$ and $\tilde{\alpha}_2$ in this situation.

Definition 2.4. Let $\tilde{\alpha} = ([a^-,\ a^+],\ [b^-,\ b^+])$ be an IVIFV, where $[a^-,\ a^+] \subseteq [0,\ 1]$, $[b^-,\ b^+] \subseteq [0,\ 1]$ and $a^+ + b^+ \le 1$. The proposed accuracy function H_{NWC} of the IVIFV $\tilde{\alpha}$ is defined as follows:

$$H_{NWC}(\tilde{\alpha}) = \frac{(1 - a^- + a^+)(1 - a^- - b^-) + (1 - b^- + b^+)(1 - a^+ - b^+)}{2}, \tag{3}$$

where $H_{NWC}(\tilde{\alpha}) \in [0,\ 1]$. The larger the accuracy value $H_{NWC}(\tilde{\alpha})$ of the IVIFV $\tilde{\alpha}$, the larger the IVIFV $\tilde{\alpha}$.

In this paper, we propose a new ranking method of IVIFVs based on the proposed score function S_{NWC} and the proposed accuracy function H_{NWC}, shown as follows.

Definition 2.5. Let $\tilde{\alpha}_1$ and $\tilde{\alpha}_2$ be any two IVIFVs, then

(1) If $S_{NWC}(\tilde{\alpha}_1) < S_{NWC}(\tilde{\alpha}_2)$, then $\tilde{\alpha}_1 < \tilde{\alpha}_2$.
(2) If $S_{NWC}(\tilde{\alpha}_1) = S_{NWC}(\tilde{\alpha}_2)$, then
  (1) If $H_{NWC}(\tilde{\alpha}_1) = H_{NWC}(\tilde{\alpha}_2)$, then $\tilde{\alpha}_1 = \tilde{\alpha}_2$.
  (2) If $H_{NWC}(\tilde{\alpha}_1) < H_{NWC}(\tilde{\alpha}_2)$, then $\tilde{\alpha}_1 < \tilde{\alpha}_2$.

Definition 2.6 [25]. Let $\tilde{\alpha}_1,\ \tilde{\alpha}_2,\ \ldots,$ and $\tilde{\alpha}_n$ be IVIFVs, where $\tilde{\alpha}_i = ([a_i,\ b_i],\ [c_i,\ d_i])$, $1 \le i \le n$, $[a_i,\ b_i] \subseteq [0,\ 1]$, $[c_i,\ d_i] \subseteq [0,\ 1]$ and $0 \le b_i + d_i \le 1$. The interval-valued intuitionistic fuzzy weighted averaging (IVIFWA) operator f_{WLQ} of the IVIFVs $\tilde{\alpha}_1,\ \tilde{\alpha}_2,\ \ldots,$ and $\tilde{\alpha}_n$ is defined as follows:

$$f_{WLQ}(\tilde{\alpha}_1, \tilde{\alpha}_2, \ldots, \tilde{\alpha}_n) = \left(\left[g^-, g^+\right],\ \left[h^-, h^+\right]\right), \tag{4}$$

where ω_i is the weight of IVIFV $\tilde{\alpha}_i$, $0 \le \omega_i \le 1$, $1 \le i \le n$, $\sum_{i=1}^{n} \omega_i = 1$, $g^- = \sum_{i=1}^{n} \omega_i a_i$, $g^+ = \sum_{i=1}^{n} \omega_i b_i$, $h^- = \sum_{i=1}^{n} \omega_i c_i$, $h^+ = \sum_{i=1}^{n} \omega_i d_i$, $0 \le g^- \le g^+ \le 1$, $0 \le h^- \le h^+ \le 1$ and $g^+ + h^+ \le 1$.

Definition 2.7 [27]. Let $\tilde{\alpha}_1,\ \tilde{\alpha}_2,\ \ldots,$ and $\tilde{\alpha}_n$ be IVIFVs, where $\tilde{\alpha}_i = ([a_i,\ b_i],\ [c_i,\ d_i])$, $1 \le i \le n$, $[a_i,\ b_i] \subseteq [0,\ 1]$, $[c_i,\ d_i] \subseteq [0,\ 1]$ and $0 \le b_i + d_i \le 1$. The IVIFWA operator f_X of the IVIFVs $\tilde{\alpha}_1,\ \tilde{\alpha}_2,\ \ldots,$ and $\tilde{\alpha}_n$ is defined as follows:

$$f_X(\tilde{\alpha}_1, \tilde{\alpha}_2, \ldots, \tilde{\alpha}_n) = \left(\left[g^-, g^+\right],\ \left[h^-, h^+\right]\right), \tag{5}$$

where ω_i denotes the weight of the IVIFV $\tilde{\alpha}_i$, $0 \le \omega_i \le 1$, $1 \le i \le n$, $\sum_{i=1}^{n} \omega_i = 1$, $g^- = 1 - \prod_{i=1}^{n} (1 - a_i)^{\omega_i}$, $g^+ = 1 - \prod_{i=1}^{n} (1 - b_i)^{\omega_i}$, $h^- = \prod_{i=1}^{n} c_i^{\omega_i}$, $h^+ = \prod_{i=1}^{n} d_i^{\omega_i}$, $0 \le g^- \le g^+ \le 1$, $0 \le h^- \le h^+ \le 1$ and $g^+ + h^+ \le 1$.

3. Analyze the drawbacks of Wang and Chen's MADM method

In this section, we analyze the drawbacks of Wang and Chen's MADM method [24] shown as follows:

(1) In Step 1 of Wang and Chen's MADM method [24], it uses Wang and Chen's score function S_{WC} shown in Eq. (1) to calculate the score values of the evaluating IVIFVs. Moreover, in Step 4 of Wang and Chen's MADM method, it also uses Wang and Chen's score function S_{WC} shown in Eq. (1) to calculate the transformed values of the weighted evaluating IVIFVs (WEIVIFVs). Because Wang and Chen's score function S_{WC} has the shortcoming that it cannot distinguish IVIFVs in some cases, Wang and Chen's MADM method cannot distinguish the PO of alternatives in some cases.
(2) The LP model "$\max M = \sum_{i=1}^{m} \sum_{j=1}^{n} (\omega_j{}^* \times t_{ij})$" constructed in Step 1 of Wang and Chen's MADM method [24] has the shortcoming that it will get an infinite number of solutions of the optimal weights of attributions when the summation values of some columns in the transformed decision matrix (TDM) are the same [3].

In the following, we use some examples to illustrate the drawbacks of Wang and Chen's MADM method [24].

Example 3.1. Let A_1, A_2, A_3 and A_4 be four alternatives and let C_1, C_2 and C_3 be three attributes. Assume that the IVIF weights $\tilde{\omega}_1$, $\tilde{\omega}_2$ and $\tilde{\omega}_3$ of the attributes C_1, C_2 and C_3 are shown as follows:

$$\tilde{\omega}_1 = ([0.10, 0.40], [0.20, 0.55]),$$
$$\tilde{\omega}_2 = ([0.20, 0.50], [0.15, 0.45]),$$
$$\tilde{\omega}_3 = ([0.25, 0.60], [0.15, 0.38]).$$

That is, $y_1^- = 0.10$, $y_1^+ = 0.40$, $z_1^- = 0.20$, $z_1^+ = 0.55$, $y_2^- = 0.20$, $y_2^+ = 0.50$, $z_2^- = 0.15$, $z_2^+ = 0.45$, $y_3^- = 0.25$, $y_3^+ = 0.60$, $z_3^- = 0.15$ and $z_3^+ = 0.38$. Assume that the decision matrix (DM) $\tilde{D}$ provided by the decision maker is as follows:

$$\tilde{D} = \left(\tilde{d}_{ij}\right)_{4\times 3} = \begin{matrix} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \end{matrix} \begin{matrix} C_1 & C_2 & C_3 \\ \left(\begin{matrix} ([0.40, 0.50], [0.30, 0.40]) \\ ([0.53, 0.70], [0.05, 0.10]) \\ ([0.30, 0.60], [0.30, 0.40]) \\ ([0.70, 0.80], [0.10, 0.20]) \end{matrix}\right. & \begin{matrix} ([0.40, 0.60], [0.20, 0.40]) \\ ([0.60, 0.63], [0.16, 0.30]) \\ ([0.50, 0.60], [0.30, 0.40]) \\ ([0.60, 0.70], [0.10, 0.30]) \end{matrix} & \left.\begin{matrix} ([0.10, 0.30], [0.50, 0.60]) \\ ([0.49, 0.70], [0.10, 0.20]) \\ ([0.50, 0.60], [0.10, 0.30]) \\ ([0.30, 0.40], [0.10, 0.20]) \end{matrix}\right) \end{matrix}.$$

In the following, we use Wang and Chen's MADM method [24] to obtain the PO of the alternatives A_1, A_2, A_3 and A_4, shown as follows:

Step 1: Based on Eq. (1) and the DM $\tilde{D} = (\tilde{d}_{ij})_{4\times 3} = ([a_{ij}^-,\ a_{ij}^+],\ [b_{ij}^-,\ b_{ij}^+])_{4\times 3}$, it computes the score value t_{ij} of evaluating IVIFV $\tilde{d}_{ij}$, where $t_{ij} = \frac{a_{ij}^- + a_{ij}^+ + \sqrt{a_{ij}^+ b_{ij}^+}(1 - a_{ij}^- - b_{ij}^-) + \sqrt{a_{ij}^- b_{ij}^-}(1 - a_{ij}^+ - b_{ij}^+)}{2}$, $t_{ij} \in [0,\ 1]$, $1 \le i \le 4$ and $1 \le j \le 3$, shown as follows:

$$t_{11} = 0.5344,\ t_{12} = 0.5980,\ t_{13} = 0.2960,$$
$$t_{21} = 0.6868,\ t_{22} = 0.6780,\ t_{23} = 0.6828,$$
$$t_{31} = 0.5480,\ t_{32} = 0.5990,\ t_{33} = 0.6460,$$
$$t_{41} = 0.7900,\ t_{42} = 0.7187,\ t_{43} = 0.4695.$$

Therefore, it obtains the TDM T, where

$$T = \left(t_{ij}\right)_{4\times 3} = \begin{matrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{matrix} \overset{\begin{matrix} C_1 & \quad C_2 & \quad C_3 \end{matrix}}{\begin{pmatrix} 0.5344 & 0.5980 & 0.2960 \\ 0.6868 & 0.6780 & 0.6828 \\ 0.5480 & 0.5990 & 0.6460 \\ 0.7900 & 0.7187 & 0.4695 \end{pmatrix}}.$$

Because the IVIF weights $\tilde{\omega}_1$, $\tilde{\omega}_2$ and $\tilde{\omega}_3$ of the attributes C_1, C_2 and C_3, respectively, are as follows:

$$\tilde{\omega}_1 = \left(\left[y_1^-, y_1^+\right], \left[z_1^-, z_1^+\right]\right) = ([0.10,\ 0.40],\ [0.20,\ 0.55]),$$
$$\tilde{\omega}_2 = \left(\left[y_2^-, y_2^+\right], \left[z_2^-, z_2^+\right]\right) = ([0.20,\ 0.50],\ [0.15,\ 0.45]),$$
$$\tilde{\omega}_3 = \left(\left[y_3^-, y_3^+\right], \left[z_3^-, z_3^+\right]\right) = ([0.25,\ 0.60],\ [0.15,\ 0.38]),$$

based on the obtained TDM $T = (t_{ij})_{4\times 3}$, it obtains the LP model: "$\max M = \sum_{i=1}^{4}\sum_{j=1}^{3}(\omega_j{}^* \times t_{ij})$", where $\omega_j{}^*$ is the optimal weight of attribute C_j, $1 \le j \le 3$, $0.10 \le \omega_1{}^* \le 0.80$, $0.20 \le \omega_2{}^* \le 0.85$, $0.25 \le \omega_3{}^* \le 0.85$ and $\omega_1{}^* + \omega_2{}^* + \omega_3{}^* = 1$.

Step 2: After solving the LP model "$\max M = \sum_{i=1}^{4}\sum_{j=1}^{3}(\omega_j{}^* \times t_{ij})$" obtained in Step 1, where $0.10 \le \omega_1{}^* \le 0.80$, $0.20 \le \omega_2{}^* \le 0.85$, $0.25 \le \omega_3{}^* \le 0.85$ and $\omega_1{}^* + \omega_2{}^* + \omega_3{}^* = 1$, it gets the optimal weights $\omega_1{}^*$, $\omega_2{}^*$ and $\omega_3{}^*$ of the attributes C_1, C_2 and C_3, respectively, where $\omega_1{}^* = 0.1000$, $\omega_2{}^* = 0.6500$ and $\omega_3{}^* = 0.2500$.

Step 3: Based on Eq. (5), the optimal weights $\omega_1{}^*$, $\omega_2{}^*$ and $\omega_3{}^*$ of the attributes C_1, C_2 and C_3, respectively, where $\omega_1{}^* = 0.1000$, $\omega_2{}^* = 0.6500$ and $\omega_3{}^* = 0.2500$, and the DM $\tilde{D} = (\tilde{d}_{ij})_{4\times 3} = ([a_{ij}^-,\ a_{ij}^+],\ [b_{ij}^-,\ b_{ij}^+])_{4\times 3}$, it computes the WEIVIFV $\tilde{E}_i = ([c_i^-,\ c_i^+],\ [d_i^-,\ d_i^+])$ of alternative A_i, where $c_i^- = 1 - \prod_{j=1}^{3}(1 - a_{ij}^-)^{\omega_j{}^*}$, $c_i^+ = 1 - \prod_{j=1}^{3}(1 - a_{ij}^+)^{\omega_j{}^*}$, $d_i^- = \prod_{j=1}^{3} b_{ij}^{-\omega_j{}^*}$, $d_i^+ = \prod_{j=1}^{3} b_{ij}^{+\omega_j{}^*}$, $0 \le c_i^- \le c_i^+ \le 1$, $0 \le d_i^- \le d_i^+ \le 1$, $c_i^+ + d_i^+ \le 1$, $1 \le i \le 4$ and $1 \le j \le 3$, shown as follows:

$$c_1^- = 0.3360,\ c_1^+ = 0.5296,\ d_1^- = 0.2619,\ d_1^+ = 0.4427,$$
$$c_2^- = 0.5680,\ c_2^+ = 0.6562,\ d_2^- = 0.1266,\ d_2^+ = 0.2429,$$
$$c_3^- = 0.4829,\ c_3^+ = 0.6000,\ d_3^- = 0.2280,\ d_3^+ = 0.3722,$$
$$c_4^- = 0.5530,\ c_4^+ = 0.6574,\ d_4^- = 0.1000,\ d_4^+ = 0.2603.$$

Therefore, it obtains the WEIVIFV $\tilde{E}_i$ of alternatives A_i, where $1 \le i \le 4$, shown as follows:

$$\tilde{E}_1 = \left(\left[c_1^-,\ c_1^+\right],\ \left[d_1^-,\ d_1^+\right]\right) = ([0.3360,\ 0.5296],\ [0.2619,\ 0.4427]),$$
$$\tilde{E}_2 = \left(\left[c_2^-,\ c_2^+\right],\ \left[d_2^-,\ d_2^+\right]\right) = ([0.5680,\ 0.6562],\ [0.1266,\ 0.2429]),$$
$$\tilde{E}_3 = \left(\left[c_3^-,\ c_3^+\right],\ \left[d_3^-,\ d_3^+\right]\right) = ([0.4829,\ 0.6000],\ [0.2280,\ 0.3722]),$$
$$\tilde{E}_4 = \left(\left[c_4^-,\ c_4^+\right],\ \left[d_4^-,\ d_4^+\right]\right) = ([0.5530,\ 0.6574],\ [0.1000,\ 0.2603]).$$

Step 4: Based on Eq. (1), it computes the transformed value E_i of the WEIVIFV $\tilde{E}_i = ([c_i^-,\ c_i^+],\ [d_i^-,\ d_i^+])$ of alternative A_i, where $E_i = \frac{c_{ij}^- + c_{ij}^+ + \sqrt{c_{ij}^+ d_{ij}^+}(1 - c_{ij}^- - d_{ij}^-) + \sqrt{c_{ij}^- d_{ij}^-}(1 - c_{ij}^+ - d_{ij}^+)}{2}$ and $1 \le i \le 4$, shown as follows:

$$E_1 = 0.5342,\ E_2 = 0.6866,\ E_3 = 0.6144,\ E_4 = 0.6866.$$

Because $E_2 = E_4 > E_3 > E_1$, we can see that it gets the PO "$A_2 = A_4 \succ A_3 \succ A_1$" of the alternatives A_1, A_2, A_3 and A_4. Therefore, Wang and Chen's MADM method [24] has the shortcoming that it cannot distinguish the PO of the alternatives A_2 and A_4 in this case.

Example 3.2. Let A_1, A_2 and A_3 be three alternatives and let C_1, C_2 and C_3 be three attributes. Assume that the IVIF weights $\tilde{\omega}_1$, $\tilde{\omega}_2$ and $\tilde{\omega}_3$ of the attributes C_1, C_2 and C_3 are shown as follows:

$$\tilde{\omega}_1 = ([0.25, 0.25], [0.25, 0.25]),$$
$$\tilde{\omega}_2 = ([0.35, 0.35], [0.40, 0.40]),$$
$$\tilde{\omega}_3 = ([0.30, 0.30], [0.65, 0.65]).$$

Assume that the DM $\tilde{D}$ provided by the decision maker is shown as follows:

$$\tilde{D} = \left(\tilde{d}_{ij}\right)_{3\times3} = \begin{matrix} \\ A_1 \\ A_2 \\ A_3 \end{matrix} \begin{matrix} C_1 & C_2 & C_3 \\ \left(\begin{matrix} ([0.37, 0.50], [0.14, 0.19]) \\ ([0.30, 0.36], [0.20, 0.25]) \\ ([0.15, 0.20], [0.45, 0.50]) \end{matrix}\right. & \begin{matrix} ([0.51, 0.54], [0.18, 0.28]) \\ ([0.60, 0.70], [0.20, 0.20]) \\ ([0.70, 0.75], [0.05, 0.10]) \end{matrix} & \left.\begin{matrix} ([0.11, 0.80], [0.17, 0.20]) \\ ([0.47, 0.47], [0.50, 0.50]) \\ ([0.60, 0.60], [0.30, 0.30]) \end{matrix}\right) \end{matrix}.$$

In the following, we use Wang and Chen's MADM method [24] to obtain the PO of the alternatives A_1, A_2 and A_3, shown as follows:

Step 1: Based on Eq. (1) and the DM $\tilde{D} = (\tilde{d}_{ij})_{3\times3} = ([a_{ij}^-,\ a_{ij}^+],\ [b_{ij}^-,\ b_{ij}^+])_{3\times3}$, it computes the score value t_{ij} of evaluating IVIFV $\tilde{d}_{ij}$, where $t_{ij} = \frac{a_{ij}^- + a_{ij}^+ + \sqrt{a_{ij}^+ b_{ij}^+}(1 - a_{ij}^- - b_{ij}^-) + \sqrt{a_{ij}^- b_{ij}^-}(1 - a_{ij}^+ - b_{ij}^+)}{2}$, $t_{ij} \in [0,\ 1]$, $1 \le i \le 3$ and $1 \le j \le 3$, shown as follows:

$$T = \left(t_{ij}\right)_{3\times3} = \begin{matrix} A_1 \\ A_2 \\ A_3 \end{matrix} \overset{\begin{matrix} C_1 & \quad C_2 & \quad C_3 \end{matrix}}{\begin{pmatrix} 0.5458 & 0.6125 & 0.5990 \\ 0.4528 & 0.7047 & 0.4845 \\ 0.2772 & 0.7733 & 0.6424 \end{pmatrix}}.$$

Because the IVIF weights $\tilde{\omega}_1$, $\tilde{\omega}_2$ and $\tilde{\omega}_3$ of the attributes C_1, C_2 and C_3, respectively, are as follows:

$$\tilde{\omega}_1 = \left(\left[y_1^-,\ y_1^+\right],\ \left[z_1^-,\ z_1^+\right]\right) = ([0.25,\ 0.25],\ [0.25,\ 0.25]),$$
$$\tilde{\omega}_2 = \left(\left[y_2^-,\ y_2^+\right],\ \left[z_2^-,\ z_2^+\right]\right) = ([0.35,\ 0.35],\ [0.40,\ 0.40]),$$
$$\tilde{\omega}_3 = \left(\left[y_3^-,\ y_3^+\right],\ \left[z_3^-,\ z_3^+\right]\right) = ([0.30,\ 0.30],\ [0.65,\ 0.65]),$$

based on the obtained TDM $T = (t_{ij})_{3\times3}$, it gets the LP model "$\max M = \sum_{i=1}^{3}\sum_{j=1}^{3}(\omega_j{}^* \times t_{ij})$", where $\omega_j{}^*$ is the optimal weight of attribute C_j, $1 \le j \le 3$, $0.25 \le \omega_1{}^* \le 0.75$, $0.35 \le \omega_2{}^* \le 0.60$, $0.30 \le \omega_3{}^* \le 0.35$ and $\omega_1{}^* + \omega_2{}^* + \omega_3{}^* = 1$.

Step 2: After solving the LP model "$\max M = \sum_{i=1}^{3}\sum_{j=1}^{3}(\omega_j{}^* \times t_{ij})$" obtained in Step 1, where $0.25 \le \omega_1{}^* \le 0.75$, $0.35 \le \omega_2{}^* \le 0.60$, $0.30 \le \omega_3{}^* \le 0.35$ and $\omega_1{}^* + \omega_2{}^* + \omega_3{}^* = 1$, it gets the optimal weight $\omega_1{}^*$, $\omega_2{}^*$ and $\omega_3{}^*$ of the attributes C_1, C_2 and C_3, respectively, where $\omega_1{}^* = 0.2500$, $\omega_2{}^* = 0.4500$ and $\omega_3{}^* = 0.3000$.

Step 3: Based on Eq. (5), the obtained optimal weights $\omega_1{}^*$, $\omega_2{}^*$ and $\omega_3{}^*$ of the attributes C_1, C_2 and C_3 obtained in Step 2, respectively, where $\omega_1{}^* = 0.2500$, $\omega_2{}^* = 0.4500$ and $\omega_3{}^* = 0.3000$, and the DM $\tilde{D} = (\tilde{d}_{ij})_{3\times3} = ([a_{ij}^-,\ a_{ij}^+],\ [b_{ij}^-,\ b_{ij}^+])_{3\times3}$, it computes the WEIVIFV $\tilde{E}_i = ([c_i^-,\ c_i^+],\ [d_i^-,\ d_i^+])$ of alternative A_i, where $c_i^- = 1 - \prod_{j=1}^{3}(1 - a_{ij}^-)^{\omega_j{}^*}$, $c_i^+ = 1 - \prod_{j=1}^{3}(1 - a_{ij}^+)^{\omega_j{}^*}$, $d_i^- = \prod_{j=1}^{3} b_{ij}^{-\omega_j{}^*}$, $d_i^+ = \prod_{j=1}^{3} b_{ij}^{+\omega_j{}^*}$, $0 \le c_i^- \le c_i^+ \le 1$, $0 \le d_i^- \le d_i^+ \le 1$, $0 \le c_i^+ + d_i^+ \le 1$, $1 \le i \le 3$ and $1 \le j \le 3$, shown as follows:

$$\tilde{E}_1 = \left(\left[c_1^-,\ c_1^+\right], \left[d_1^-,\ d_1^+\right]\right) = ([0.3759,\ 0.6342],\ [0.1662,\ 0.2297]),$$
$$\tilde{E}_2 = \left(\left[c_2^-,\ c_2^+\right], \left[d_2^-, d_2^+\right]\right) = ([0.4994,\ 0.5699],\ [0.2633,\ 0.2784]),$$
$$\tilde{E}_3 = \left(\left[c_3^-, c_3^+\right], \left[d_3^-,\ d_3^+\right]\right) = ([0.5757,\ 0.6150],\ [0.1482,\ 0.2079]).$$

Step 4: Based on Eq. (1), it computes the transformed value E_i of the WEIVIFV $\tilde{E}_i = ([c_i^-,\ c_i^+],\ [d_i^-,\ d_i^+])$ of alternative A_i, where $E_i = \frac{c_{ij}^- + c_{ij}^+ + \sqrt{c_{ij}^+ d_{ij}^+}(1 - c_{ij}^- - d_{ij}^-) + \sqrt{c_{ij}^- d_{ij}^-}(1 - c_{ij}^+ - d_{ij}^+)}{2}$, $E_i \in [0,\ 1]$ and $1 \le i \le 3$, shown as follows:

$$E_1 = 0.6094,\ E_2 = 0.6094,\ E_3 = 0.6706.$$

Because $E_3 > E_1 = E_2$, it gets the PO "$A_3 \succ A_1 = A_2$" of the alternatives A_1, A_2 and A_3. Therefore, Wang and Chen's MADM method [24] has the shortcoming that it cannot distinguish the PO between the alternatives A_1 and A_2 in this case.

Example 3.3. Let A_1, A_2, A_3 and A_4 be four alternatives and let C_1, C_2 and C_3 be three attributes. Assume that the IVIF weights $\tilde{\omega}_1$, $\tilde{\omega}_2$ and $\tilde{\omega}_3$ of the attributes C_1, C_2 and C_3 are shown as follows:

$$\tilde{\omega}_1 = ([0.10,\ 0.40],\ [0.20,\ 0.55]),$$

$$\tilde{\omega}_2 = ([0.20,\ 0.50],\ [0.15,\ 0.45]),$$
$$\tilde{\omega}_3 = ([0.25,\ 0.60],\ [0.15,\ 0.38]).$$

Assume that the DM $\tilde{D}$ provided by the decision maker is shown as follows:

$$\tilde{D} = \left(\tilde{d}_{ij}\right)_{4\times 3} = \begin{matrix} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \end{matrix} \begin{matrix} C_1 & C_2 & C_3 \\ \left(\begin{matrix} ([0.40,\ 0.50],\ [0.30,\ 0.40]) & ([0.41,\ 0.60],\ [0.10,\ 0.30]) & ([0.32,\ 0.60],\ [0.27,\ 0.40]) \\ ([0.30,\ 0.70],\ [0.05,\ 0.10]) & ([0.50,\ 0.60],\ [0.10,\ 0.30]) & ([0.42,\ 0.63],\ [0.12,\ 0.21]) \\ ([0.32,\ 0.60],\ [0.20,\ 0.40]) & ([0.40,\ 0.50],\ [0.20,\ 0.40]) & ([0.40,\ 0.60],\ [0.10,\ 0.33]) \\ ([0.60,\ 0.70],\ [0.15,\ 0.20]) & ([0.40,\ 0.60],\ [0.14,\ 0.20]) & ([0.41,\ 0.60],\ [0.11,\ 0.29]) \end{matrix}\right) \end{matrix}.$$

In the following, we use Wang and Chen's MADM method [24] to obtain the PO of the alternatives A_1, A_2, A_3 and A_4, shown as follows:

Step 1: Based on Eq. (1) and the DM $\tilde{D} = (\tilde{d}_{ij})_{4\times 3} = ([a_{ij}^-,\ a_{ij}^+],\ [b_{ij}^-,\ b_{ij}^+])_{4\times 3}$, it computes the score value t_{ij} of evaluating IVIFV $\tilde{d}_{ij}$, where $t_{ij} = \frac{a_{ij}^- + a_{ij}^+ + \sqrt{a_{ij}^+ b_{ij}^+}(1 - a_{ij}^- - b_{ij}^-) + \sqrt{a_{ij}^- b_{ij}^-}(1 - a_{ij}^+ - b_{ij}^+)}{2}$, $t_{ij} \in [0,\ 1]$, $1 \le i \le 4$ and $1 \le j \le 3$, shown as follows:

$$T = \left(t_{ij}\right)_{4\times 3} = \begin{matrix} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \end{matrix} \begin{matrix} C_1 \quad\quad C_2 \quad\quad C_3 \\ \left(\begin{matrix} 0.5344 & 0.6191 & 0.5604 \\ 0.5982 & 0.6460 & 0.6266 \\ 0.5776 & 0.5536 & 0.6182 \\ 0.7118 & 0.6033 & 0.6168 \end{matrix}\right) \end{matrix}.$$

Because the IVIF weights $\tilde{\omega}_1$, $\tilde{\omega}_2$ and $\tilde{\omega}_3$ of the attributes C_1, C_2 and C_3, respectively, are shown as follows:

$$\tilde{\omega}_1 = \left(\left[y_1^-,\ y_1^+\right],\ \left[z_1^-,\ z_1^+\right]\right) = ([0.10,\ 0.40],\ [0.20,\ 0.55]),$$
$$\tilde{\omega}_2 = \left(\left[y_2^-,\ y_2^+\right],\ \left[z_2^-,\ z_2^+\right]\right) = ([0.20,\ 0.50],\ [0.15,\ 0.45]),$$
$$\tilde{\omega}_3 = \left(\left[y_3^-,\ y_3^+\right],\ \left[z_3^-,\ z_3^+\right]\right) = ([0.25,\ 0.60],\ [0.15,\ 0.38]),$$

based on the obtained TDM $T = (t_{ij})_{4\times 3}$, it gets the LP model "$\max M = \sum_{i=1}^{4}\sum_{j=1}^{3}(\omega_j{}^* \times t_{ij})$", where $\omega_j{}^*$ is the optimal weight of attribute C_j, where $1 \le j \le 3$, $0.10 \le \omega_1{}^* \le 0.80$, $0.20 \le \omega_2{}^* \le 0.85$, $0.25 \le \omega_3{}^* \le 0.85$ and $\omega_1{}^* + \omega_2{}^* + \omega_3{}^* = 1$.

Step 2: From the obtained TDM $T = (t_{ij})_{4\times 3}$, we can see that $t_{11} + t_{21} + t_{31} + t_{41} = t_{12} + t_{22} + t_{32} + t_{42} = t_{13} + t_{23} + t_{33} + t_{43} = 2.4220$. Therefore, the LP model "$\max M = \sum_{i=1}^{4}\sum_{j=1}^{3}(\omega_j{}^* \times t_{ij})$" becomes $\omega_1{}^* \times (t_{11} + t_{21} + t_{31} + t_{41}) + \omega_2{}^* \times (t_{12} + t_{22} + t_{32} + t_{42}) + \omega_3{}^* \times (t_{13} + t_{23} + t_{33} + t_{43}) = \omega_1{}^* \times 2.4220 + \omega_2{}^* \times 2.4220 + \omega_3{}^* \times 2.4220 = (\omega_1{}^* + \omega_2{}^* + \omega_3{}^*) \times 2.4220 = 2.4220$, where $\omega_1{}^* + \omega_2{}^* + \omega_3{}^* = 1$. That is, in this LP model, it will obtain an infinite number of optimal weights $\omega_1{}^*$, $\omega_2{}^*$ and $\omega_3{}^*$ of the attributes C_1, C_2 and C_3, respectively, which satisfies $\omega_1{}^* + \omega_2{}^* + \omega_3{}^* = 1$. For example, it can obtain the following two sets of optimal weights $\omega_1{}^*$, $\omega_2{}^*$ and $\omega_3{}^*$ of the attributes C_1, C_2 and C_3, respectively:

(1) $\omega_1{}^* = 0.3000$, $\omega_2{}^* = 0.4000$, $\omega_3{}^* = 0.3000$,
(2) $\omega_1{}^* = 0.1000$, $\omega_2{}^* = 0.2000$, $\omega_3{}^* = 0.7000$,

which satisfies $\omega_1{}^* + \omega_2{}^* + \omega_3{}^* = 1$.

Step 3: For the optimal weights $\omega_1{}^* = 0.3000$, $\omega_2{}^* = 0.4000$ and $\omega_3{}^* = 0.3000$, based on Eq. (5) and the DM $\tilde{D} = (\tilde{d}_{ij})_{4\times 3} = ([a_{ij}^-,\ a_{ij}^+],\ [b_{ij}^-,\ b_{ij}^+])_{4\times 3}$, it computes the WEIVIFV $\tilde{E}_i = ([c_i^-,\ c_i^+],\ [d_i^-,\ d_i^+])$ of alternative A_i, where $c_i^- = 1 - \prod_{j=1}^{3}(1 - a_{ij}^-)^{\omega_j{}^*}$, $c_i^+ = 1 - \prod_{j=1}^{3}(1 - a_{ij}^+)^{\omega_j{}^*}$, $d_i^- = \prod_{j=1}^{3} b_{ij}^{-\omega_j{}^*}$, $d_i^+ = \prod_{j=1}^{3} b_{ij}^{+\omega_j{}^*}$, $0 \le c_i^- \le c_i^+ \le 1$, $0 \le d_i^- \le d_i^+ \le 1$, $0 \le c_i^+ + d_i^+ \le 1$ and $1 \le i \le 4$, shown as follows:

$$\tilde{E}_1 = \left(\left[c_1^-,\ c_1^+\right],\ \left[d_1^-,\ d_1^+\right]\right) = ([0.3812,\ 0.5723],\ [0.1873,\ 0.3565]),$$
$$\tilde{E}_2 = \left(\left[c_2^-,\ c_2^+\right],\ \left[d_2^-,\ d_2^+\right]\right) = ([0.4217,\ 0.6416],\ [0.0858,\ 0.1939]),$$
$$\tilde{E}_3 = \left(\left[c_3^-,\ c_3^+\right],\ \left[d_3^-,\ d_3^+\right]\right) = ([0.3770,\ 0.5627],\ [0.1625,\ 0.3776]),$$
$$\tilde{E}_4 = \left(\left[c_4^-,\ c_4^+\right],\ \left[d_4^-,\ d_4^+\right]\right) = ([0.4714,\ 0.6331],\ [0.1330,\ 0.2236]).$$

In the same way, for the optimal weights $\omega_1{}^* = 0.1000$, $\omega_2{}^* = 0.2000$ and $\omega_3{}^* = 0.7000$, it gets

$$\tilde{E}_1 = \left(\left[c_1^-,\ c_1^+\right],\ \left[d_1^-,\ d_1^+\right]\right) = ([0.3473,\ 0.5910],\ [0.2237,\ 0.3776]),$$
$$\tilde{E}_2 = \left(\left[c_2^-,\ c_2^+\right],\ \left[d_2^-,\ d_2^+\right]\right) = ([0.4263,\ 0.6320],\ [0.1060,\ 0.2094]),$$
$$\tilde{E}_3 = \left(\left[c_3^-,\ c_3^+\right],\ \left[d_3^-,\ d_3^+\right]\right) = ([0.3924,\ 0.5817],\ [0.1231,\ 0.3496]),$$
$$\tilde{E}_4 = \left(\left[c_4^-,\ c_4^+\right],\ \left[d_4^-,\ d_4^+\right]\right) = ([0.4306,\ 0.6113],\ [0.1191,\ 0.2594]).$$

Step 4: For the optimal weights $\omega_1{}^* = 0.3000$, $\omega_2{}^* = 0.4000$ and $\omega_3{}^* = 0.3000$, based on Eq. (1), it computes the transformed value E_i of the WEIVIFV $\tilde{E}_i = ([c_i^-,\ c_i^+],\ [d_i^-,\ d_i^+])$ of alternative A_i, where $E_i =$

$\frac{c_{ij}^- + c_{ij}^+ + \sqrt{c_{ij}^+ d_{ij}^+}(1 - c_{ij}^- - d_{ij}^-) + \sqrt{c_{ij}^- d_{ij}^-}(1 - c_{ij}^+ - d_{ij}^+)}{2}$ and $1 \le i \le 4$, shown as follows:

$$E_1 = 0.5837,\ E_2 = 0.6341,\ E_3 = 0.5834,\ E_4 = 0.6446.$$

Because $E_4 > E_2 > E_1 > E_3$, it gets the PO "$A_4 \succ A_2 \succ A_1 \succ A_3$" of the alternatives A_1, A_2, A_3 and A_4. In the same way, for the optimal weights $\omega_1{}^* = 0.1000$, $\omega_2{}^* = 0.2000$ and $\omega_3{}^* = 0.7000$, it gets

$$E_1 = 0.5748,\ E_2 = 0.6311,\ E_3 = 0.6039,\ E_4 = 0.6253.$$

Because $E_2 > E_4 > E_3 > E_1$, it gets the PO "$A_2 \succ A_4 \succ A_3 \succ A_1$" of the alternatives A_1, A_2, A_3 and A_4. In other words, Wang and Chen's MADM method [24] obtains two different POs of the alternatives A_1, A_2, A_3 and A_4 in this case, which is unreasonable.

4. A new MADM method using the LP methodology and the proposed new score function and the proposed new accuracy function of IVIFVs

Assume that $A_1, A_2, \ldots,$ and A_m are m alternatives and assume that $C_1, C_2, \ldots,$ and C_n are n attributes. Let the DM $\tilde{D} = (\tilde{d}_{ij})_{m\times n} = ([a_{ij}^-,\ a_{ij}^+],\ [b_{ij}^-,\ b_{ij}^+])_{m\times n}$ provided by the decision maker be represented by IVIFVs. Let the weight of attribute C_j provided by the decision maker be represented by an IVIF weight $\tilde{\omega}_j$, where $\tilde{\omega}_j = ([y_j^-,\ y_j^+],\ [z_j^-,\ z_j^+])$ and $1 \le j \le n$. The proposed MADM method is shown as follows:

Step 1: Based on Eq. (2) and the DM $\tilde{D} = (\tilde{d}_{ij})_{m\times n} = ([a_{ij}^-,\ a_{ij}^+],\ [b_{ij}^-,\ b_{ij}^+])_{m\times n}$, build the transformed decision matrix (TDM) $T = (t_{ij})_{m\times n}$, where $t_{ij} = \frac{(a_{ij}^- + a_{ij}^+)(a_{ij}^- + b_{ij}^-) - (b_{ij}^- + b_{ij}^+)(a_{ij}^+ + b_{ij}^+)}{2}$, $t_{ij} \in [-1,\ 1]$, $1 \le i \le m$ and $1 \le j \le n$. Based on the TDM $T = (t_{ij})_{m\times n}$, construct the following LP model [4]:

$$\max M = \sum_{i=1}^{m}\sum_{j=1}^{n}\left(\omega_j{}^* \times t_{ij}\right), \tag{6}$$

where $\omega_j{}^*$ is the optimal weight of attribute C_j, $y_j^- \le \omega_j{}^* \le 1 - z_j^-$, $1 \le j \le n$ and $\sum_{j=1}^n \omega_j{}^* = 1$.

Step 2: If the summation values of the elements in each column of the TDM $T = (t_{ij})_{m\times n}$ are different, then solve the LP model obtained in Step 1 to obtain the optimal weight $\omega_j{}^*$ of attribute C_j, where $1 \le j \le n$. Otherwise, if there are s columns in the obtained TDM $T = (t_{ij})_{m\times n}$ whose summation values for these s columns are the same and there are $(n - s)$ columns in the obtained TDM $T = (t_{ij})_{m\times n}$ whose summation values of the columns are different, where $2 \le s \le n$, $\sum_{i=1}^m t_{i1} = \sum_{i=1}^m t_{i2} = \cdots = \sum_{i=1}^m t_{is} = r$ and $\sum_{i=1}^m t_{i(s+1)} \ne \sum_{i=1}^m t_{i(s+2)} \ne \cdots \ne \sum_{i=1}^m t_{in}$, then do the following sub-steps:

Step 2.1: Compute the standard deviation σ_j of the values at the jth column of the obtained TDM $T = (t_{ij})_{m\times n}$, where $1 \le j \le s$, shown as follows:

$$\sigma_j = \sqrt{\frac{\sum_{i=1}^m \left(t_{ij} - \mu\right)^2}{m}}, \tag{7}$$

where $\mu = \frac{1}{m}\sum_{i=1}^m t_{i1} = \frac{1}{m}\sum_{i=1}^m t_{i2} = \cdots = \frac{1}{m}\sum_{i=1}^m t_{is} = \frac{r}{m}$.

Step 2.2: Sort the obtained standard deviations $\sigma_1, \sigma_2, \cdots,$ and σ_s in an ascending sequence. Assume that the ascending sequence of the obtained standard deviations $\sigma_1, \sigma_2, \ldots,$ and σ_s is $\sigma_1 \le \sigma_2 \le \cdots \le \sigma_s$.

Step 2.3: Add the small delta values $\delta_1, \delta_2, \ldots,$ and δ_s to the elements $t_{11}, t_{12}, \ldots,$ and t_{1s} of the first column, the second column, $\cdots$, and the sth column of the obtained TDM $T = (t_{ij})_{m\times n}$, respectively, where $2 \le s \le n$ (Note: In this paper, we let the small delta values $\delta_1 = 0.0001$, $\delta_2 = 0.0002$, $\cdots$, and $\delta_s = 0.0001 \times s$), to get the modified TDM $T' = (t'_{ij})_{m\times n}$, where the other elements in the modified TDM $T' = (t'_{ij})_{m\times n}$ are the same as the ones of the TDM $T = (t_{ij})_{m\times n}$.

Step 2.4: Based on the modified TDM $T' = (t'_{ij})_{m\times n}$, reconstruct the following LP model:

$$\max M = \sum_{i=1}^{m}\sum_{j=1}^{n}\left(\omega_j{}^* \times t'_{ij}\right), \tag{8}$$

where $\omega_j{}^*$ is the optimal weight of attribute C_j, $y_j^- \le \omega_j{}^* \le 1 - z_j^-$, $1 \le j \le n$ and $\sum_{j=1}^n \omega_j{}^* = 1$.

Step 2.5: Solve the LP model shown in Eq. (8) to get the optimal weight $\omega_j{}^*$ of attribute C_j, where $1 \le j \le n$.

Step 3: Based on Eq. (4), the obtained optimal weight $\omega_j{}^*$ of the attribute C_j, where $1 \le j \le n$, and the DM $\tilde{D} = (\tilde{d}_{ij})_{m\times n} = ([a_{ij}^-,\ a_{ij}^+],\ [b_{ij}^-,\ b_{ij}^+])_{m\times n}$, aggregate the evaluating IVIFVs $\tilde{d}_{i1}, \tilde{d}_{i2}, \cdots,$ and $\tilde{d}_{in}$ into the WEIVIFV $\tilde{E}_i = ([c_i^-,\ c_i^+],\ [d_i^-,\ d_i^+])$ of alternative A_i, where $c_i^- = \sum_{j=1}^n \omega_j{}^* a_{ij}^-$, $c_i^+ = \sum_{j=1}^n \omega_j{}^* a_{ij}^+$, $d_i^- = \sum_{j=1}^n \omega_j{}^* b_{ij}^-$, $d_i^+ = \sum_{j=1}^n \omega_j{}^* b_{ij}^+$, $0 \le c_i^- \le c_i^+ \le 1$, $0 \le d_i^- \le d_i^+ \le 1$, $0 \le c_i^+ + d_i^+ \le 1$, $0 < \omega_j{}^* \le 1$, $1 \le i \le m$, $1 \le j \le n$ and $\sum_{j=1}^n \omega_j{}^* = 1$.

Step 4: Based on Eq. (2), calculate the transformed value E_i of the WEIVIFV $\tilde{E}_i = ([c_i^-, c_i^+], [d_i^-, d_i^+])$ of alternative A_i, where $E_i = \frac{(c_i^- + c_i^+)(c_i^- + d_i^-) - (d_i^- + d_i^+)(c_i^+ + d_i^+)}{2}$, $E_i \in [-1, 1]$ and $1 \le i \le m$. The larger the transformed value E_i, the better the PO of alternative A_i, where $1 \le i \le m$. If $E_k = E_l$, where $1 \le k \le m$, $1 \le l \le m$ and $k \ne l$, then based on Eq. (3), calculate the accuracy value F_k of the WEIVIFV $\tilde{E}_k = ([c_k^-, c_k^+], [d_k^-, d_k^+])$ of alternative A_k, where $F_k = \frac{(1-c_k^- + c_k^+)(1 - c_k^- - d_k^-) + (1-d_k^- + d_k^+)(1-c_k^+ - d_k^+)}{2}$, $F_k \in [0, 1]$ and $1 \le k \le m$. In the same way, compute the accuracy value F_l of the WEIVIFV $\tilde{E}_l = ([c_l^-, c_l^+], [d_l^-, d_l^+])$ of alternative A_l, where $F_l = \frac{(1-c_l^- + c_l^+)(1 - c_l^- - d_l^-) + (1-d_l^- + d_l^+)(1-c_l^+ - d_l^+)}{2}$, $F_l \in [0, 1]$, $1 \le l \le m$ and $k \ne l$. If $F_k > F_l$, then the PO of alternatives A_k and A_l is: $A_k \succ A_l$, where $1 \le k \le m$, $1 \le l \le m$ and $k \ne l$; if $F_k = F_l$, then the PO of atternatives A_k and A_l is $A_k = A_l$, where $1 \le k \le m$, $1 \le l \le m$ and $k \ne l$; if $F_k < F_l$, then the PO of alternatives A_k and A_l is: $A_k \prec A_l$, where $1 \le k \le m$, $1 \le l \le m$ and $k \ne l$.

Example 4.1. The same assumptions as those in Example 3.1. The procedure of the proposed MADM method is shown as follows:

Step 1: Based on Eq. (2) and the DM $\tilde{D} = (\tilde{d}_{ij})_{4\times3} = ([a_{ij}^-, a_{ij}^+], [b_{ij}^-, b_{ij}^+])_{4\times3}$, we can get the score value t_{ij} of evaluating IVIFV $\tilde{d}_{ij}$, where $t_{ij} = \frac{(a_{ij}^- + a_{ij}^+)(a_{ij}^- + b_{ij}^-) - (b_{ij}^- + b_{ij}^+)(a_{ij}^+ + b_{ij}^+)}{2}$, $t_{ij} \in [-1, 1]$, $1 \le i \le 4$ and $1 \le j \le 3$, shown as follows:

$$t_{11} = 0,\ t_{12} = 0,\ t_{13} = -0.3750,$$
$$t_{21} = 0.2967,\ t_{22} = 0.2535,\ t_{23} = 0.2160,$$
$$t_{31} = -0.0800,\ t_{32} = 0.0900,\ t_{33} = 0.1500,$$
$$t_{41} = 0.4500,\ t_{42} = 0.2550,\ t_{43} = 0.0500.$$

Therefore, we can obtain the TDM T, where

$$T = (t_{ij})_{4\times3} = \begin{matrix} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \end{matrix} \begin{matrix} C_1 & C_2 & C_3 \\ \end{matrix} \begin{pmatrix} 0 & 0 & -0.3750 \\ 0.2967 & 0.2535 & 0.2160 \\ -0.0800 & 0.0900 & 0.1500 \\ 0.4500 & 0.2550 & 0.0500 \end{pmatrix}.$$

Because the IVIF weights $\tilde{\omega}_1$, $\tilde{\omega}_2$ and $\tilde{\omega}_3$ of the attributes C_1, C_2 and C_3, respectively, are shown as follows:

$$\tilde{\omega}_1 = \left(\left[y_1^-, y_1^+\right], \left[z_1^-, z_1^+\right]\right) = ([0.10, 0.40], [0.20, 0.55]),$$

$$\tilde{\omega}_2 = \left(\left[y_2^-, y_2^+\right], \left[z_2^-, z_2^+\right]\right) = ([0.20, 0.50], [0.15, 0.45]),$$

$$\tilde{\omega}_3 = \left(\left[y_3^-, y_3^+\right], \left[z_3^-, z_3^+\right]\right) = ([0.25, 0.60], [0.15, 0.38]),$$

based on the obtained TDM $T = (t_{ij})_{4\times3}$, we can get the LP model "$\max M = \sum_{i=1}^4 \sum_{j=1}^3 (\omega_j{}^* \times t_{ij})$", where $\omega_1{}^*$, $\omega_2{}^*$ and $\omega_3{}^*$ are the optimal weights of the attributes C_1, C_2 and C_3, respectively, $0.10 \le \omega_1{}^* \le 0.80$, $0.20 \le \omega_2{}^* \le 0.85$, $0.25 \le \omega_3{}^* \le 0.85$ and $\omega_1{}^* + \omega_2{}^* + \omega_3{}^* = 1$.

Step 2: Because the summation values of the elements in each column of the TDM are different, where $\sum_{i=1}^4 t_{i1} = 0.6667$, $\sum_{i=1}^4 t_{i2} = 0.5985$ and $\sum_{i=1}^4 t_{i3} = 0.0410$, after solving the LP model "$\max M = \sum_{i=1}^4 \sum_{j=1}^3 (\omega_j{}^* \times t_{ij})$" obtained in Step 1, where $0.10 \le \omega_1{}^* \le 0.80$, $0.20 \le \omega_2{}^* \le 0.85$, $0.25 \le \omega_3{}^* \le 0.85$ and $\omega_1{}^* + \omega_2{}^* + \omega_3{}^* = 1$, we can get the optimal weights $\omega_1{}^*$, $\omega_2{}^*$ and $\omega_3{}^*$ of the attributes C_1, C_2 and C_3, respectively, where $\omega_1{}^* = 0.5500$, $\omega_2{}^* = 0.2000$ and $\omega_3{}^* = 0.2500$.

Step 3: Based on Eq. (4), the DM $\tilde{D} = (\tilde{d}_{ij})_{4\times3} = ([a_{ij}^-, a_{ij}^+], [b_{ij}^-, b_{ij}^+])_{4\times3}$ and the optimal weights $\omega_1{}^*$, $\omega_2{}^*$ and $\omega_3{}^*$ of the attributes C_1, C_2 and C_3 obtained in Step 2, respectively, where $\omega_1{}^* = 0.5500$, $\omega_2{}^* = 0.2000$ and $\omega_3{}^* = 0.2500$, we can obtain the WEIVIFV $\tilde{E}_i = ([c_i^-, c_i^+], [d_i^-, d_i^+])$ of alternative A_i, where $c_i^- = \sum_{j=1}^3 \omega_j{}^* a_{ij}^-$, $c_i^+ = \sum_{j=1}^3 \omega_j{}^* a_{ij}^+$, $d_i^- = \sum_{j=1}^3 \omega_j{}^* b_{ij}^-$, $d_i^+ = \sum_{j=1}^3 \omega_j{}^* b_{ij}^+$, $0 \le c_i^- \le c_i^+ \le 1$, $0 \le d_i^- \le d_i^+ \le 1$, $0 \le c_i^+ + d_i^+ \le 1$ and $1 \le i \le 4$, shown as follows:

$$\tilde{E}_1 = \left(\left[c_1^-, c_1^+\right], \left[d_1^-, d_1^+\right]\right) = ([0.3250, 0.4700], [0.3300, 0.4500]),$$
$$\tilde{E}_2 = \left(\left[c_2^-, c_2^+\right], \left[d_2^-, d_2^+\right]\right) = ([0.5340, 0.6860], [0.0845, 0.1650]),$$
$$\tilde{E}_3 = \left(\left[c_3^-, c_3^+\right], \left[d_3^-, d_3^+\right]\right) = ([0.3900, 0.6000], [0.2500, 0.3750]),$$
$$\tilde{E}_4 = \left(\left[c_4^-, c_4^+\right], \left[d_4^-, d_4^+\right]\right) = ([0.5800, 0.6800], [0.1000, 0.2200]).$$

Step 4: Based on Eq. (2), we can compute the transformed value E_i of the WEIVIFV $\tilde{E}_i = ([c_i^-, c_i^+], [d_i^-, d_i^+])$ of alternative A_i, where $E_i = \frac{(c_i^- + c_i^+)(c_i^- + d_i^-) - (d_i^- + d_i^+)(c_i^+ + d_i^+)}{2}$, $E_i \in [-1, 1]$ and $1 \le i \le 4$, shown as follows:

$$E_1 = -0.0984,\ E_2 = 0.2711,\ E_3 = 0.0121,\ E_4 = 0.2844.$$

Because $E_4 > E_2 > E_3 > E_1$, we can see that the PO of the alternatives A_1, A_2, A_3 and A_4 is: $A_4 \succ A_2 \succ A_3 \succ A_1$. It is obvious that the proposed MADM method can distinguish the PO between the alternatives A_2 and A_4, whereas Wang and Chen's MADM method [24] cannot distinguish the PO between the alternatives A_2 and A_4, as shown in Example 3.1. Therefore, the proposed MADM method can overcome the shortcoming of Wang and Chen's MADM method [24] in this case.

Example 4.2. Let A_1, A_2 and A_3 be three alternatives and let C_1, C_2 and C_3 be three attributes. Assume that the IVIF weights $\tilde{\omega}_1$, $\tilde{\omega}_2$ and $\tilde{\omega}_3$ of the attributes C_1, C_2 and C_3 are shown as follows:

$$\tilde{\omega}_1 = ([0.25,\ 0.25],\ [0.25,\ 0.25]),$$
$$\tilde{\omega}_2 = ([0.35,\ 0.35],\ [0.40,\ 0.40]),$$
$$\tilde{\omega}_3 = ([0.30,\ 0.30],\ [0.65,\ 0.65]).$$

Assume that the DM $\tilde{D}$ provided by the decision maker is shown as follows:

$$\tilde{D} = \left(\tilde{d}_{ij}\right)_{3\times 3} = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \end{array} \begin{array}{c} \begin{array}{ccc} \quad C_1 \quad & \quad C_2 \quad & \quad C_3 \quad \end{array} \\ \begin{pmatrix} ([0.45,\ 0.66],\ [0.15,\ 0.20]) & ([0.50,\ 0.70],\ [0.13,\ 0.28]) & ([0.30,\ 0.80],\ [0.16,\ 0.20]) \\ ([0.30,\ 0.48],\ [0.20,\ 0.25]) & ([0.60,\ 0.70],\ [0.20,\ 0.20]) & ([0.45,\ 0.47],\ [0.50,\ 0.50]) \\ ([0.15,\ 0.20],\ [0.45,\ 0.50]) & ([0.70,\ 0.75],\ [0.05,\ 0.10]) & ([0.60,\ 0.60],\ [0.30,\ 0.30]) \end{pmatrix} \end{array}.$$

The procedure of the proposed MADM method is shown as follows:

Step 1: Based on Eq. (2) and the DM $\tilde{D} = (\tilde{d}_{ij})_{3\times 3} = ([a_{ij}^-,\ a_{ij}^+],\ [b_{ij}^-,\ b_{ij}^+])_{3\times 3}$, we can compute the score value t_{ij} of evaluating IVIFV $\tilde{d}_{ij}$, where $t_{ij} = \frac{(a_{ij}^- + a_{ij}^+)(a_{ij}^- + b_{ij}^-) - (b_{ij}^- + b_{ij}^+)(a_{ij}^+ + b_{ij}^+)}{2}$, $t_{ij} \in [-1,\ 1]$, $1 \le i \le 3$ and $1 \le j \le 3$, shown as follows:

$$T = \left(t_{ij}\right)_{3\times 3} = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \end{array} \begin{array}{c} \begin{array}{ccc} C_1 & C_2 & C_3 \end{array} \\ \begin{pmatrix} 0.1825 & 0.1771 & 0.0730 \\ 0.0308 & 0.3400 & -0.0480 \\ -0.2275 & 0.4800 & 0.2700 \end{pmatrix} \end{array}.$$

Because the IVIF weights $\tilde{\omega}_1$, $\tilde{\omega}_2$ and $\tilde{\omega}_3$ of the attributes C_1, C_2 and C_3, respectively, are shown as follows:

$$\tilde{\omega}_1 = \left(\left[y_1^-,\ y_1^+\right],\ \left[z_1^-,\ z_1^+\right]\right) = ([0.25,\ 0.25],\ [0.25,\ 0.25]),$$
$$\tilde{\omega}_2 = \left(\left[y_2^-,\ y_2^+\right],\ \left[z_2^-,\ z_2^+\right]\right) = ([0.35,\ 0.35],\ [0.40,\ 0.40]),$$
$$\tilde{\omega}_3 = \left(\left[y_3^-,\ y_3^+\right],\ \left[z_3^-,\ z_3^+\right]\right) = ([0.30,\ 0.30],\ [0.65,\ 0.65]).$$

based on the obtained TDM $T = (t_{ij})_{3\times 3}$, we can get the LP model "$\max M = \sum_{i=1}^{3}\sum_{j=1}^{3}(\omega_j{}^* \times t_{ij})$", where $\omega_1{}^*$, $\omega_2{}^*$ and $\omega_3{}^*$ are the optimal weights of the attributes C_1, C_2 and C_3, respectively, $0.25 \le \omega_1{}^* \le 0.75$, $0.35 \le \omega_2{}^* \le 0.60$, $0.30 \le \omega_3{}^* \le 0.35$ and $\omega_1{}^* + \omega_2{}^* + \omega_3{}^* = 1$.

Step 2: Because the summation values of the elements in each column of the TDM are different, where $\sum_{i=1}^{3} t_{i1} = -0.0142$, $\sum_{i=1}^{3} t_{i2} = 0.9971$ and $\sum_{i=1}^{3} t_{i3} = 0.2950$, after solving the LP model "$\max M = \sum_{i=1}^{3}\sum_{j=1}^{3}(\omega_j{}^* \times t_{ij})$" obtained in Step 1, where $0.25 \le \omega_1{}^* \le 0.75$, $0.35 \le \omega_2{}^* \le 0.60$, $0.30 \le \omega_3{}^* \le 0.35$ and $\omega_1{}^* + \omega_2{}^* + \omega_3{}^* = 1$, we can get $\omega_1{}^* = 0.2500$, $\omega_2{}^* = 0.4500$ and $\omega_3{}^* = 0.3000$.

Step 3: Based on Eq. (4), the DM $\tilde{D} = (\tilde{d}_{ij})_{3\times 3} = ([a_{ij}^-,\ a_{ij}^+],\ [b_{ij}^-,\ b_{ij}^+])_{3\times 3}$ and the optimal weights $\omega_1{}^*$, $\omega_2{}^*$ and $\omega_3{}^*$ of the attributes C_1, C_2 and C_3 obtained in Step 2, respectively, where $\omega_1{}^* = 0.2500$, $\omega_2{}^* = 0.4500$ and $\omega_3{}^* = 0.3000$, we can get the WEIVIFV $\tilde{E}_i = ([c_i^-,\ c_i^+],\ [d_i^-,\ d_i^+])$ of alternative A_i, where $c_i^- = \sum_{j=1}^{3}\omega_j{}^* a_{ij}^-$, $c_i^+ = \sum_{j=1}^{3}\omega_j{}^* a_{ij}^+$, $d_i^- = \sum_{j=1}^{3}\omega_j{}^* b_{ij}^-$, $d_i^+ = \sum_{j=1}^{3}\omega_j{}^* b_{ij}^+$, $0 \le c_i^- \le c_i^+ \le 1$, $0 \le d_i^- \le d_i^+ \le 1$, $0 \le c_i^+ + d_i^+ \le 1$ and $1 \le i \le 3$, shown as follows:

$$\tilde{E}_1 = \left(\left[c_1^-,\ c_1^+\right],\ \left[d_1^-,\ d_1^+\right]\right) = ([0.4275,\ 0.7200],\ [0.1440,\ 0.2360]),$$
$$\tilde{E}_2 = \left(\left[c_2^-,\ c_2^+\right],\ \left[d_2^-,\ d_2^+\right]\right) = ([0.4800,\ 0.5760],\ [0.2900,\ 0.3025]),$$
$$\tilde{E}_3 = \left(\left[c_3^-,\ c_3^+\right],\ \left[d_3^-,\ d_3^+\right]\right) = ([0.5325,\ 0.5675],\ [0.2250,\ 0.2600]).$$

Step 4: Based on Eq. (2), we can compute the transformed value E_i of the WEIVIFV $\tilde{E}_i = ([c_i^-,\ c_i^+],\ [d_i^-,\ d_i^+])$ of alternative A_i, where $E_i = \frac{(c_i^- + c_i^+)(c_i^- + d_i^-) - (d_i^- + d_i^+)(c_i^+ + d_i^+)}{2}$, $E_i \in [-1,\ 1]$ and $1 \le i \le 3$, shown as follows:

$$E_1 = 0.1463,\ E_2 = 0.1463,\ E_3 = 0.2160.$$

Because $E_3 > E_1 = E_2$, based on Eq. (3), we can compute the accuracy value F_i of the WEIVIFV $\tilde{E}_i = ([c_i^-,\ c_i^+],\ [d_i^-,\ d_i^+])$ of alternative A_i, where $F_k = \frac{(1-c_k^- + c_k^+)(1-c_k^- - d_k^-) + (1-d_k^- + d_k^+)(1-c_k^+ - d_k^+)}{2}$, $F_k \in [0,\ 1]$ and $1 \le k \le 2$. That is, $F_1 = 0.3009$ and $F_2 = 0.1875$. Because $F_1 > F_2$, we can get the PO of the alternatives: $A_3 \succ A_1 \succ A_2$.

Example 4.3. Let A_1, A_2, A_3 and A_4 be four alternatives and let C_1, C_2 and C_3 be three attributes. Assume that the IVIF weights $\tilde{\omega}_1$, $\tilde{\omega}_2$ and $\tilde{\omega}_3$ of the attributes C_1, C_2 and C_3 are shown as follows:

$$\tilde{\omega}_1 = ([0.10,\ 0.40],\ [0.20,\ 0.55]),$$

$\tilde{\omega}_2 = ([0.20, 0.50], [0.15, 0.45]),$
$\tilde{\omega}_3 = ([0.25, 0.60], [0.15, 0.38]).$

Assume that the DM $\tilde{D}$ provided by the decision maker is shown as follows:

$$\tilde{D} = \left(\tilde{d_{ij}}\right)_{4\times3} = \begin{matrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{matrix} \overset{\begin{matrix} C_1 & & & C_2 & & & C_3 \end{matrix}}{\begin{pmatrix} ([0.32, 0.51], [0.34, 0.43]) & ([0.41, 0.60], [0.10, 0.30]) & ([0.41, 0.60], [0.19, 0.40]) \\ ([0.32, 0.75], [0.03, 0.11]) & ([0.51, 0.60], [0.10, 0.30]) & ([0.42, 0.70], [0.10, 0.21]) \\ ([0.42, 0.60], [0.29, 0.40]) & ([0.40, 0.50], [0.20, 0.40]) & ([0.45, 0.60], [0.10, 0.33]) \\ ([0.61, 0.70], [0.08, 0.22]) & ([0.40, 0.60], [0.14, 0.20]) & ([0.45, 0.70], [0.10, 0.29]) \end{pmatrix}}.$$

The procedure of the proposed MADM method is shown as follows:

Step 1: Based on Eq. (2) and the DM $\tilde{D} = (\tilde{d_{ij}})_{4\times3}$, we can construct the TDM $T = (t_{ij})_{4\times3}$, shown as follows:

$$T = \left(t_{ij}\right)_{4\times3} = \begin{matrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{matrix} \overset{\begin{matrix} C_1 & C_2 & C_3 \end{matrix}}{\begin{pmatrix} -0.0880 & 0.0776 & 0.0080 \\ 0.1271 & 0.1585 & 0.1501 \\ 0.0171 & 0 & 0.0888 \\ 0.3139 & 0.1340 & 0.1232 \end{pmatrix}}.$$

Because the IVIF weights $\tilde{\omega}_1$, $\tilde{\omega}_2$ and $\tilde{\omega}_3$ of the attributes C_1, C_2 and C_3, respectively, are shown as follows:

$$\tilde{\omega}_1 = \left(\left[y_1^-, y_1^+\right], \left[z_1^-, z_1^+\right]\right) = ([0.10, 0.40], [0.20, 0.55]),$$
$$\tilde{\omega}_2 = \left(\left[y_2^-, y_2^+\right], \left[z_2^-, z_2^+\right]\right) = ([0.20, 0.50], [0.15, 0.45]),$$
$$\tilde{\omega}_3 = \left(\left[y_3^-, y_3^+\right], \left[z_3^-, z_3^+\right]\right) = ([0.25, 0.60], [0.15, 0.38]),$$

based on the obtained TDM $T = (t_{ij})_{4\times3}$, we can get the LP model "$\max M = \sum_{i=1}^{4}\sum_{j=1}^{3}(\omega_j{}^* \times t_{ij})$", where $\omega_1{}^*$, $\omega_2{}^*$ and $\omega_3{}^*$ are the optimal weights of the attributes C_1, C_2 and C_3, respectively, $0.10 \le \omega_1{}^* \le 0.80$, $0.20 \le \omega_2{}^* \le 0.85$, $0.25 \le \omega_3{}^* \le 0.85$ and $\omega_1{}^* + \omega_2{}^* + \omega_3{}^* = 1$.

Step 2: Because the summation values of the elements in each column of the TDM are the same, where $\sum_{i=1}^{4} t_{i1} = \sum_{i=1}^{4} t_{i2} = \sum_{i=1}^{4} t_{i3} = 0.3701$, we do the following sub-steps:

Step 2.1: After calculating the standard deviation σ_j of the jth column of the obtained TDM $T = (t_{ij})_{4\times3}$, where $1 \le j \le 3$, we can get

$$\sigma_1 = 0.2609, \quad \sigma_2 = 0.1364, \quad \sigma_3 = 0.1234.$$

Step 2.2: After sorting the standard deviations σ_1, σ_2 and σ_3 in an ascending sequence, we can get $\sigma_3 < \sigma_2 < \sigma_1$.

Step 2.3: After adding the delta values 0.0001, 0.0002 and 0.0003 to the first elements of t_{13}, t_{12} and t_{11} of the third column, the second column and the first column of the TDM $T = (t_{ij})_{4\times3}$, respectively (Note: In this paper, we let $\delta_1 = 0.0001$, let $\delta_2 = 0.0002$ and let $\delta_3 = 0.0003$), we can get the modified TDM $T' = (t'_{ij})_{4\times3}$, where $t'_{13} = t_{13} + 0.0001 = -0.0877$, $t'_{12} = t_{12} + 0.0002 = 0.0778$, $t'_{11} = t_{11} + 0.0003 = 0.0081$ and the other elements in the modified TDM $T' = (t'_{ij})_{4\times3}$ are the same as the ones of the TDM $T = (t_{ij})_{4\times3}$, shown as follows:

$$T' = \left(t'_{ij}\right)_{4\times3} = \begin{matrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{matrix} \overset{\begin{matrix} C_1 & C_2 & C_3 \end{matrix}}{\begin{pmatrix} -0.0877 & 0.0778 & 0.0081 \\ 0.1271 & 0.1585 & 0.1501 \\ 0.0171 & 0 & 0.0888 \\ 0.3139 & 0.1340 & 0.1232 \end{pmatrix}}.$$

Step 2.4: Based on the modified TDM $T' = (t'_{ij})_{4\times3}$, we can get the LP model "$\max M = \sum_{i=1}^{4}\sum_{j=1}^{3}(\omega_j{}^* \times t'_{ij})$", where $\omega_j{}^*$ is the optimal weight of attribute C_j, where $1 \le j \le 3$, $0.10 \le \omega_1{}^* \le 0.80$, $0.20 \le \omega_2{}^* \le 0.85$, $0.25 \le \omega_3{}^* \le 0.85$ and $\sum_{j=1}^{3}\omega_j{}^* = 1$. After solving the LP model "$\max M = \sum_{i=1}^{4}\sum_{j=1}^{3}(\omega_j{}^* \times t'_{ij})$", where $0.10 \le \omega_1{}^* \le 0.80$, $0.20 \le \omega_2{}^* \le 0.85$, $0.25 \le \omega_3{}^* \le 0.85$ and $\omega_1{}^* + \omega_2{}^* + \omega_3{}^* = 1$, we can get the optimal weights $\omega_1{}^*$, $\omega_2{}^*$ and $\omega_3{}^*$ of the attributes C_1, C_2 and C_3, respectively, where $\omega_1{}^* = 0.5500$, $\omega_2{}^* = 0.2000$ and $\omega_3{}^* = 0.2500$.

Step 3: Based on Eq. (4), the DM $\tilde{D} = (\tilde{d_{ij}})_{4\times3} = ([a_{ij}^-, a_{ij}^+], [b_{ij}^-, b_{ij}^+])_{4\times3}$ and the optimal weights $\omega_1{}^*$, $\omega_2{}^*$ and $\omega_3{}^*$ of the attributes C_1, C_2 and C_3 obtained in Step 2, respectively, where $\omega_1{}^* = 0.5500$, $\omega_2{}^* = 0.2000$ and $\omega_3{}^* = 0.2500$, we can obtain the WEIVIFV $\tilde{E}_i = ([c_i^-, c_i^+], [d_i^-, d_i^+])$ of alternative A_i, where $c_i^- = \sum_{j=1}^{3}\omega_j{}^* a_{ij}^-$, $c_i^+ = \sum_{j=1}^{3}\omega_j{}^* a_{ij}^+$, $d_i^- = \sum_{j=1}^{3}\omega_j{}^* b_{ij}^-$, $d_i^+ = \sum_{j=1}^{3}\omega_j{}^* b_{ij}^+$, $0 \le c_i^- \le c_i^+ \le 1$, $0 \le d_i^- \le d_i^+ \le 1$, $0 \le c_i^+ + d_i^+ \le 1$ and $1 \le i \le 4$, shown as follows:

$$\tilde{E}_1 = \left(\left[c_1^-, c_1^+\right], \left[d_1^-, d_1^+\right]\right) = ([0.3605, 0.5505], [0.2545, 0.3965]),$$
$$\tilde{E}_2 = \left(\left[c_2^-, c_2^+\right], \left[d_2^-, d_2^+\right]\right) = ([0.3830, 0.7075], [0.0615, 0.1730]),$$
$$\tilde{E}_3 = \left(\left[c_3^-, c_3^+\right], \left[d_3^-, d_3^+\right]\right) = ([0.4235, 0.5800], [0.2245, 0.3825]),$$

$$\tilde{E}_4 = \left(\left[c_4^-, c_4^+\right], \left[d_4^-, d_4^+\right]\right) = ([0.5280, 0.6800], [0.0970, 0.2335]).$$

Step 4: Based on Eq. (2), we can calculate the transformed value E_i of the WEIVIFV $\tilde{E}_i = ([c_i^-, c_i^+], [d_i^-, d_i^+])$ of alternative A_i, where $E_i = \frac{(c_i^- + c_i^+)(c_i^- + d_i^-) - (d_i^- + d_i^+)(c_i^+ + d_i^+)}{2}$, $E_i \in [-1, 1]$ and $1 \le i \le 4$, shown as follows:

$$E_1 = -0.0281, E_2 = 0.1391, E_3 = 0.0330, E_4 = 0.2265.$$

Because $E_4 > E_2 > E_3 > E_1$, we can see that the PO of the alternatives A_1, A_2, A_3 and A_4 is: $A_4 \succ A_2 \succ A_3 \succ A_1$.

5. Conclusions

In this paper, we have proposed a new MADM method using the LP methodology and the proposed new score function and the proposed new accuracy function of IVIFVs to overcome the shortcomings of Wang and Chen's MADM method [24] for dealing with IVIF MADM problems. The proposed MADM method can overcome the shortcomings of Wang and Chen's MADM method [24], which has the shortcomings that it cannot distinguish the preference orders (POs) of alternatives in some circumstances and it gets different POs of alternatives due to the fact that it gets an infinite number of solutions of the optimal weights of attributes when the summation values of some columns in the transformed decision matrix (TDM) are the same. Granular computing [15–17] is a problem solving method that can be used to deal with MADM problems [10,13,14,19,23] and multiple attribute group decision making (MAGDM) problems [8,9,11,12,18,20]. In granular computing, decision makers can express their evaluating values more flexibly by using fuzzy sets, rough sets, vague sets or intervals. It is worth of future research to use granular computing techniques to further develop MADM methods and MAGDM methods.

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